Natural Convection
• Air flow caused by a difference in temperature
• The air close to the plate is heated and thereby assumes a lower density
• The air closest to the plate starts to rise

Natural Convection
• The air flow gives rise to heat exchange

Laminar or turbulent flow?
• Looking at the flow close to the plate, we can distinguish between: laminar and turbulent flow.

Laminar flow
• Laminar flow is:
  - "orderly"
  - follows known theory
  - … and is therefore predictable!
  • Accurate results can be achieved by numerical solution of the differential equations governing the fluid flow and heat transfer!

Turbulent flow
• Turbulent flow is:
  - Chaotic, stochastisc
  • Turbulence is small fluctuations in velocity, pressure, temperature
  - theory to describe this completely exists, but cannot be applied due to lack of computational resources
  • It is however possible to model the effects that the turbulence have on friction (shear stresses), and heat transfer numerically
  • Large computer power is needed to do this!

Laminar or turbulent flow
• Laminar flow generally yields poorer heat transfer compared to turbulent flow
• The reason for this is that the turbulence causes a mixing of the fluid close to the plate.
• This mixing gives rise to better heat transfer due to thinner boundary layers close to the plate.
Heat transfer from a plane plate

- Newton’s "law" of cooling
  \[ Q = h \cdot A \cdot \Delta t \]
  \[ q^* = \frac{Q}{A} = h \cdot \Delta t \]

- If we can calculate the heat transfer coefficient, h, then we can calculate the heat dissipation, Q

Heat transfer from a vertical plane plate

- Instead of solving the differential equations (and model the turbulence), results from experimental investigations have been presented in form of correlations like:

  \[
  \text{Nu} = \frac{h \cdot L}{\lambda} = \frac{C \cdot Ra^{3/2}}{L^{3/2}}
  \]

  - Rayleigh number, Ra
  - Characteristic length, L
  - Thermal conductivity, \( \lambda \)
  - Nusselt number

- Solving for the heat transfer coefficient:
  \[
  Nu = \frac{h \cdot L}{\lambda} = \frac{C \cdot Ra^{n}}{L^{n}}
  \]

  - Rayleigh number, Ra
  - Characteristic length, L
  - Thermo-physical properties

- Observe!

Heat transfer from a vertical plane plate

- Rayleigh number, Ra (or product Gr·Pr)

  \[
  Ra = \frac{Gr}{Pr} = \frac{g \cdot \beta \cdot \Delta t \cdot L}{\nu \cdot \mu}
  \]

  - Rayleigh number, Ra
  - Product of Gr (gravitational number) and Pr (Prandtl number)

- Natural constants and fluid properties

  - Temperature, \( t \)
  - Density, \( \rho \)
  - Thermal conductivity, \( k \)
  - Specific heat, \( c_p \)
  - Dynamic viscosity, \( \mu \)
  - Kinematic viscosity, \( \nu \)
  - Prandtl number, \( Pr \)
  - Gravitational acceleration, \( g \)
  - Temperature difference, \( \Delta t \)
  - Length, \( L \)

- Observe!

Properties for air

- This table is attached to the handouts
Laminar or turbulent?

- **Laminar flow if** $10^4 < Ra_L < 10^8$
  
  => $C = 0.56$ and $n = 1/4$

- **Turbulent flow if** $10^8 < Ra_L < 10^{12}$
  
  => $C = 0.13$ and $n = 1/3$

\[
\text{Nu} = \frac{h \cdot L}{\lambda} = C \cdot Ra_L^\alpha \Rightarrow h = \frac{\lambda}{L} \cdot C \left( Ra_L \cdot \Delta T \cdot L \right)^{\frac{\alpha}{4}}
\]

\[
\text{Nu}_L = 0.56 \cdot Ra_L^{\frac{1}{4}} \Rightarrow h = \lambda \cdot 0.56 \cdot Ra_L^{\frac{1}{4}} \left( \Delta T \right)^{\frac{1}{4}}
\]

\[
\text{Nu}_L = 0.13 \cdot Ra_L^{\frac{1}{3}} \Rightarrow h = \lambda \cdot 0.13 \cdot Ra_L^{\frac{1}{3}} \left( \Delta T \right)^{\frac{1}{3}}
\]

Example

Calculate the heat dissipation by natural convection from a vertical plane plate with surface temperature of 60 °C. The height of the plate is 0.5 m and its width is 1 m. The ambient air is 20 °C. Heat is dissipated from both sides of the plate.

Example

Calculate film temperature, $t_{\text{film}}$

\[t_{\text{film}} = (60 + 20)/2 = 40 ^\circ C\]

Properties for air

\[
\begin{array}{ccccccccc}
\text{Temp.} & \text{Density} & \text{Spec. heat} & \text{Kin visc.} & \text{Dyn. visc.} & \text{Pr} & \text{C}_{ra} \\
\text{C} & \text{kg/m}^3 & \text{kcal/kg} \cdot \text{K} & \text{m}^2/\text{s} & \text{m}^2/\text{s} & - & \text{m}^2/\text{K} \\
-196 & 960 & 1.82 & 0.2 & 0.196 \cdot 10^5 & 188 \cdot 10^5 & 1.712 \\
\end{array}
\]

Air at atmospheric pressure $p = 1.013 \text{ bar (1 atm)}$, thermal properties:

- $150$ $2.897$ $1.016$ $0.0113$ $2.9 \cdot 10^{-8}$ $8.5 \cdot 10^{-8}$ $0.766$ $7068 \cdot 10^{-8}$
- $100$ $2.346$ $1.008$ $0.0159$ $5.7 \cdot 10^{-8}$ $17.7 \cdot 10^{-8}$ $0.748$ $1296 \cdot 10^{-8}$
- $50$ $1.384$ $1.006$ $0.0200$ $9.2 \cdot 10^{-8}$ $44.9 \cdot 10^{-8}$ $0.721$ $379 \cdot 10^{-8}$
- $0$ $1.295$ $1.006$ $0.0241$ $13.3 \cdot 10^{-8}$ $72.7 \cdot 10^{-8}$ $0.715$ $146 \cdot 10^{-8}$
- $+20$ $1.205$ $1.006$ $0.0277$ $15.1 \cdot 10^{-8}$ $18.1 \cdot 10^{-8}$ $0.710$ $553 \cdot 10^{-8}$
- $+40$ $1.127$ $1.007$ $0.0323$ $16.0 \cdot 10^{-8}$ $19.1 \cdot 10^{-8}$ $0.705$ $771 \cdot 10^{-8}$
- $+60$ $1.060$ $1.008$ $0.0368$ $18.9 \cdot 10^{-8}$ $25.0 \cdot 10^{-8}$ $0.701$ $583 \cdot 10^{-8}$

\[t_{\text{film}} = 40 ^\circ C \Rightarrow C_{ra} = 77.1 \cdot 10^6 \text{ m}^{-2} \cdot \text{K}^{-1}; \lambda = 0.0273 \text{ W/(m} \cdot \text{K})\]

Boundary conditions

- So far, we have considered the temperature as constant.
- Another boundary condition could be to have a constant heat flux, $q^\prime$ (heat dissipation per surface area)

Example

Calculate film temperature, $t_{\text{film}}$

\[t_{\text{film}} = (60 + 20)/2 = 40 ^\circ C\]

\[C_{ra} = 77.1 \cdot 10^6 \text{ m}^{-2} \cdot \text{K}^{-1}; \lambda = 0.0273 \text{ W/(m} \cdot \text{K})\]

\[Ra_L = C_{ra} \cdot \Delta T \cdot L = 77.1 \cdot 10^6 \cdot (60 - 20) \cdot 0.5^3 = 3.85 \cdot 10^9\]

Turbulent! => $C = 0.13; n = 1/3$

\[h = \frac{\lambda}{L} \cdot C \cdot Ra_L^{\frac{1}{3}} = \frac{0.0273}{0.5} \cdot 0.13 \cdot 3.85 \cdot 10^{12} = 5.16 \text{ W/(m}^2 \cdot \text{K})\]

\[Q = h \cdot A \cdot \Delta T = 5.16 \cdot 2 \cdot 0.5 \cdot (60 - 20) = 206.6 \text{ W}\]

Example:

Calculate heat dissipation by natural convection from a vertical plane plate with surface temperature of 60 °C. The height of the plate is 0.5 m and its width is 1 m. The ambient air is 20 °C. Heat is dissipated from both sides of the plate.

Example:

Calculate film temperature, $t_{\text{film}}$

\[t_{\text{film}} = (60 + 20)/2 = 40 ^\circ C\]

Properties for air

\[
\begin{array}{ccccccccc}
\text{Temp.} & \text{Density} & \text{Spec. heat} & \text{Kin visc.} & \text{Dyn. visc.} & \text{Pr} & \text{C}_{ra} \\
\text{C} & \text{kg/m}^3 & \text{kcal/kg} \cdot \text{K} & \text{m}^2/\text{s} & \text{m}^2/\text{s} & - & \text{m}^2/\text{K} \\
-196 & 960 & 1.82 & 0.2 & 0.196 \cdot 10^5 & 188 \cdot 10^5 & 1.712 \\
\end{array}
\]

Air at atmospheric pressure $p = 1.013 \text{ bar (1 atm)}$, thermal properties:

- $150$ $2.897$ $1.016$ $0.0113$ $2.9 \cdot 10^{-8}$ $8.5 \cdot 10^{-8}$ $0.766$ $7068 \cdot 10^{-8}$
- $100$ $2.346$ $1.008$ $0.0159$ $5.7 \cdot 10^{-8}$ $17.7 \cdot 10^{-8}$ $0.748$ $1296 \cdot 10^{-8}$
- $50$ $1.384$ $1.006$ $0.0200$ $9.2 \cdot 10^{-8}$ $44.9 \cdot 10^{-8}$ $0.721$ $379 \cdot 10^{-8}$
- $0$ $1.295$ $1.006$ $0.0241$ $13.3 \cdot 10^{-8}$ $72.7 \cdot 10^{-8}$ $0.715$ $146 \cdot 10^{-8}$
- $+20$ $1.205$ $1.006$ $0.0277$ $15.1 \cdot 10^{-8}$ $18.1 \cdot 10^{-8}$ $0.710$ $553 \cdot 10^{-8}$
- $+40$ $1.127$ $1.007$ $0.0323$ $16.0 \cdot 10^{-8}$ $19.1 \cdot 10^{-8}$ $0.705$ $771 \cdot 10^{-8}$
- $+60$ $1.060$ $1.008$ $0.0368$ $18.9 \cdot 10^{-8}$ $25.0 \cdot 10^{-8}$ $0.701$ $583 \cdot 10^{-8}$

\[t_{\text{film}} = 40 ^\circ C \Rightarrow C_{ra} = 77.1 \cdot 10^6 \text{ m}^{-2} \cdot \text{K}^{-1}; \lambda = 0.0273 \text{ W/(m} \cdot \text{K})\]
Boundary conditions

- Heat flux can be written
  \[ q^* = \frac{Q}{A} = h \cdot \Delta t \Rightarrow \Delta t = \frac{q^*}{h} \]

- Heat transfer coefficient can be calculated as
  \[ h = \frac{\lambda}{L} \cdot C \left( \frac{C_{Ra} \cdot q^*}{h \cdot L^3} \right)^n \]

- Equation for \( \Delta t \) is inserted in the equation for \( h \):
  \[ h = \frac{\lambda}{L} \cdot C \left( \frac{C_{Ra} \cdot q^*}{h \cdot L^3} \right)^n \]

Randvillkor

- Which can be re-written:
  \[ h = \frac{\lambda}{L} \cdot C \left( \frac{C_{Ra} \cdot q^*}{h \cdot L^3} \right)^n = \left( \frac{\lambda}{L} \cdot C \cdot C_{Ra}^n \right)^{\frac{1}{n+1}} \cdot \left( \frac{q^* \cdot L^3}{\lambda} \right)^{\frac{n}{n+1}} \]

- For the laminar case with \( C = 0,56 \) and \( n = 1/4 \):
  \[ h = \left( \frac{\lambda}{L} \cdot 0,56 \cdot C_{Ra}^{\frac{1}{4}} \right)^{\frac{1}{5}} \left( \frac{q^*}{L} \right)^{\frac{1}{5}} \]

- For the turbulent case with \( C = 0,13 \) and \( n = 1/3 \):
  \[ h = \left( \frac{\lambda}{L} \cdot 0,13 \cdot C_{Ra}^{\frac{1}{3}} \right)^{\frac{1}{4}} \cdot q^*^{\frac{1}{4}} \]